

Олимпиада «Физтех» по математике

10 класс, 2015 год, вариант 1

1. Решите уравнение

$$\frac{|\cos x| - \cos 3x}{\cos x \sin 2x} = \frac{2}{\sqrt{3}}.$$

$$\mathbb{Z} \ni u, \frac{\pi}{4} + 2\pi n, \frac{\pi}{2} + 2\pi n, \frac{3\pi}{4} + 2\pi n, \frac{5\pi}{4} + 2\pi n, \frac{3\pi}{2} + 2\pi n, \frac{7\pi}{4} + 2\pi n, \pi + 2\pi n, \frac{5\pi}{2} + 2\pi n, \frac{9\pi}{4} + 2\pi n \in \mathbb{Z}$$

2. Дан правильный 20-угольник. Найдите количество четвёрок вершин этого 20-угольника, являющихся вершинами выпуклых четырёхугольников, у которых есть хотя бы одна пара параллельных сторон.

792

3. Найдите количество натуральных чисел k , не превосходящих 291000 и таких, что $k^2 - 1$ делится нацело на 291.

4007

4. Решите систему

$$\begin{cases} x^2 + y^2 \leq 2, \\ 81x^4 - 18x^2y^2 + y^4 - 360x^2 - 40y^2 + 400 = 0. \end{cases}$$

$$\left(\frac{\sqrt{2}}{1} - i, \frac{\sqrt{2}}{3} - i \right), \left(\frac{\sqrt{2}}{1} - i, \frac{\sqrt{2}}{3} \right), \left(\frac{\sqrt{2}}{1}, \frac{\sqrt{2}}{3} - i \right), \left(\frac{\sqrt{2}}{1}, \frac{\sqrt{2}}{3} \right)$$

5. Найдите все значения параметра b , для каждого из которых найдётся число a такое, что система

$$\begin{cases} x = |y - b| + \frac{3}{b}, \\ x^2 + y^2 + 32 = a(2y - a) + 12x \end{cases}$$

имеет хотя бы одно решение (x, y) .

$$(\infty + i\frac{\pi}{2}] \cap (0; \infty -)$$

6. Четырёхугольник $ABCD$ вписан в окружность с центром O . Две окружности Ω_1 и Ω_2 равных радиусов с центрами O_1 и O_2 вписаны в углы BAD и BCD соответственно, при этом первая касается стороны AD в точке K , а вторая касается стороны BC в точке T .

а) Найдите радиус окружности Ω_1 , если $AK = 2$, $CT = 8$.

б) Пусть дополнительно известно, что точка O_2 является центром окружности, описанной около треугольника BOC . Найдите угол BDC .

$$\frac{\pi}{4}; \frac{\pi}{2}; \frac{3\pi}{4}; \frac{\pi}{3}; \frac{2\pi}{3}; \frac{\pi}{6}; \frac{5\pi}{6}; \frac{\pi}{12}; \frac{11\pi}{12}; \frac{\pi}{5}; \frac{2\pi}{5}; \frac{3\pi}{5}; \frac{4\pi}{5}; \frac{\pi}{7}; \frac{2\pi}{7}; \frac{3\pi}{7}; \frac{4\pi}{7}; \frac{5\pi}{7}; \frac{6\pi}{7}; \frac{\pi}{8}; \frac{3\pi}{8}; \frac{5\pi}{8}; \frac{7\pi}{8}; \frac{\pi}{9}; \frac{2\pi}{9}; \frac{4\pi}{9}; \frac{5\pi}{9}; \frac{7\pi}{9}; \frac{8\pi}{9}; \frac{\pi}{10}; \frac{3\pi}{10}; \frac{7\pi}{10}; \frac{9\pi}{10}; \frac{\pi}{11}; \frac{2\pi}{11}; \frac{3\pi}{11}; \frac{4\pi}{11}; \frac{5\pi}{11}; \frac{6\pi}{11}; \frac{7\pi}{11}; \frac{8\pi}{11}; \frac{9\pi}{11}; \frac{10\pi}{11}; \frac{\pi}{12}; \frac{5\pi}{12}; \frac{7\pi}{12}; \frac{11\pi}{12}; \frac{\pi}{15}; \frac{2\pi}{15}; \frac{4\pi}{15}; \frac{7\pi}{15}; \frac{8\pi}{15}; \frac{11\pi}{15}; \frac{14\pi}{15}; \frac{\pi}{18}; \frac{5\pi}{18}; \frac{7\pi}{18}; \frac{11\pi}{18}; \frac{13\pi}{18}; \frac{\pi}{20}; \frac{3\pi}{20}; 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